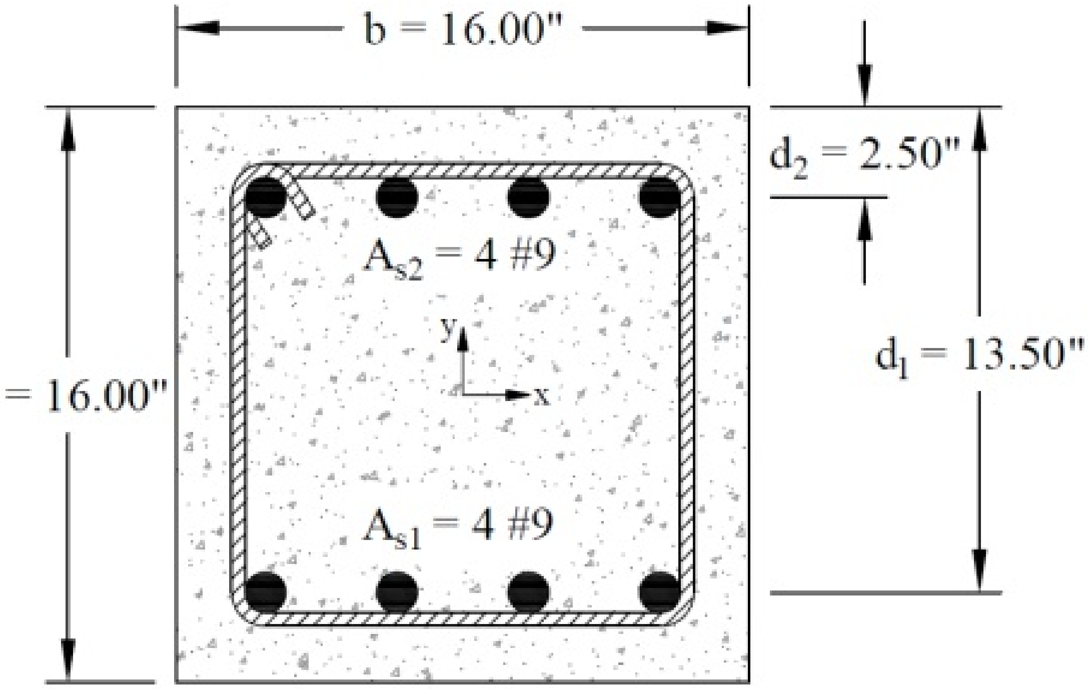
# Pilar - Flexão Composta Obliqua 1

## FLEXÃO COMPOSTA OBLIQUA

Neste exemplo, será comparada a curva de interação gerada pelo TQS para um pilar conforme dados abaixo, de forma a comparar com os cálculos manuais do ACI 318-19 e os gerados por outro software.

O exemplo foi obtido da página web da empresa Structure Point que foi desenvolvida pela PCA (Portland Cement Association) e é muito conhecida. O exemplo tem várias referências e usa a norma ACI 318-19.

Dados:




f´c = 5ksify = 60 ksil = 18 ft

h = 16 inb = 16 in c= 2.5 in

Reforço superior e inferior: 4#9

Cálculos manuais:

## 1. Maximum Compression

### 1.1. Nominal axial compressive strength at zero eccentricity

$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$$ ***ACI 318-19 (22.4.2.2)***

$$P_o = 0.85 \times 5000 \times (16 \times 16 - 8 \times 1.00) + 60000 \times 8 \times 1.00 = 1{,}534.0 \text{ kips}$$

### 1.2. Factored axial compressive strength at zero eccentricity

Since this column is a tied column with steel strain in compression:

$$\phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_o = 0.65 \times 1{,}534.0 = 997.1 \text{ kips}$$

Since the section is regular (symmetrical) about the x-axis, the moment capacity associated with the maximum axial compressive strength is equal to zero.

$$M_o = \phi M_o = 0.00 \text{ kip.ft}$$

### 1.3. Maximum (allowable) factored axial compressive strength

$$\phi P_{n,max} = 0.80 \times \phi P_o = 0.80 \times 997 = 797.7 \text{ kips}$$ ***ACI 318-19 (Table 22.4.2.1)***

## 2. Bar Stress Near Tension Face Equal to Zero, ($\varepsilon_s = f_s = 0$)

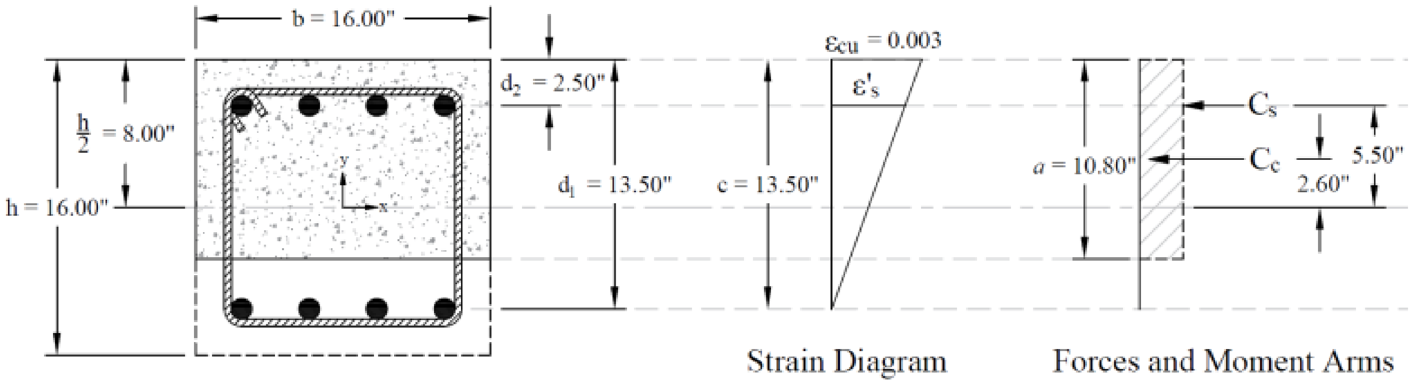



Figure 3 – Strains, Forces, and Moment Arms ($\varepsilon_t = f_s = 0$)

Strain $\varepsilon_s$ is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. ***ACI 318-19 (10.7.5.2.1 and 2)***

### 2.1. c, *a*, and strains in the reinforcement

$$c = d_1 = 13.5 \text{ in.}$$

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

***ACI 318-19 (22.2.2.4.2)***

$$a = \beta_1 \times c = 0.80 \times 13.5 = 10.80 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$a$ = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s = 0$$

$$\therefore \phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon'_c = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (13.50 - 2.5) \times \frac{0.003}{13.50} = 0.00244 \text{ (Compression)} > \varepsilon_y = \frac{F_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

### 2.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5{,}000 \times 10.80 \times 16 = 734.4 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = 0 \text{ psi} \rightarrow \mathrm{T}_s = f_s \times A_{s1} = 0 \text{ kip}$$

Since $\varepsilon_s' > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$$\therefore f_s' = f_y = 60{,}000 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ($ab$) used to compute $C_c$. As a result, it is necessary to subtract $0.85f_c'$ from $f_s'$ before computing $C_s$:

$$\mathrm{C}_s = (f_s' - 0.85 f_c') \times A_{s2} = (60{,}000 - 0.85 \times 5{,}000) \times 4 = 223 \text{ kip}$$

### 2.3. $\phi P_n$ and $\phi M_n$

$$P_n = C_c + C_s - T_s = 734.4 + 223 - 0 = 957.4 \text{ kip}$$

$$\phi P_n = 0.65 \times 957 = 622.3 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 734.4 \times \left(\frac{16}{2} - \frac{10.80}{2}\right) + 223 \times \left(\frac{16}{2} - 2.5\right) + 0 \times \left(13.50 - \frac{16}{2}\right) = 261.33 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 261 = 169.86 \text{ kip.ft}$$

## 3. Bar Stress Near Tension Face Equal to $0.5 f_y$, ($f_s = 0.5 f_y$)

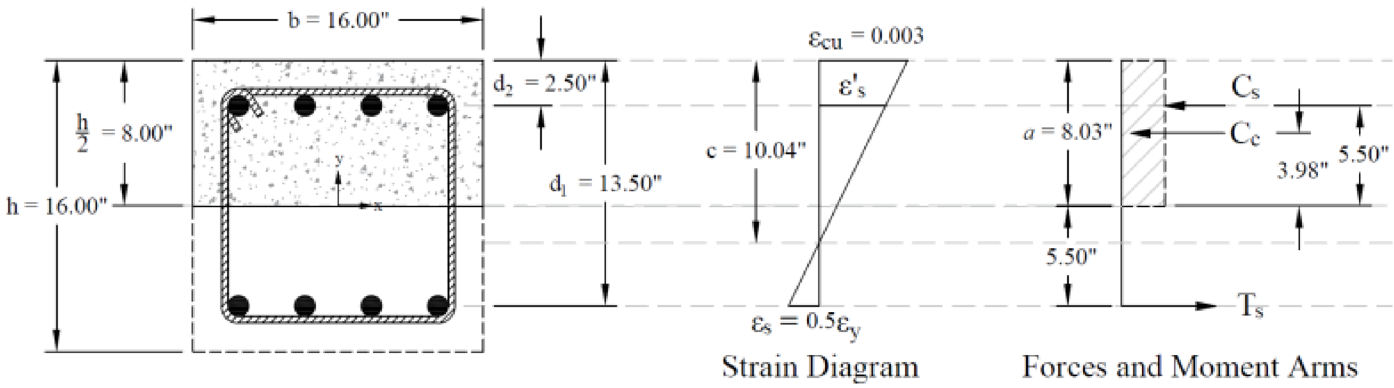



Figure 4 – Strains, Forces, and Moment Arms ($f_s = 0.5\ f_y$)

### 3.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.00207}{2} = 0.00103 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$$\therefore \phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00103 + 0.003} \times 0.003 = 10.04 \text{ in.}$$

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

***ACI 318-19 (22.2.2.4.2)***

$$a = \beta_1 \times c = 0.80 \times 10.04 = 8.03 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (10.04 - 2.5) \times \frac{0.003}{10.04} = 0.00225 \text{ (Compression)} > \varepsilon_y$$

### 3.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5{,}000 \times 8.03 \times 16 = 546.1 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = \varepsilon_s \times E_s = 0.00103 \times 29{,}000{,}000 = 30{,}000 \text{ psi}$$

$T_s = f_s \times A_{s1} = 30,000 \times 4 = 120$ kip

Since $\varepsilon_s' > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$\therefore f_s' = f_y = 60,000$ psi

The area of the reinforcement in this layer has been included in the area ($ab$) used to compute $C_c$. As a result, it is necessary to subtract $0.85f_c$' from $f_s$' before computing $C_s$:

$$C_s = (f_s' - 0.85 f_c') \times A_{s2} = (60,000 - 0.85 \times 5,000) \times 4 = 223 \text{ kip}$$

3.3. $\phi P_n$ and $\phi M_n$

$$P_n = C_c + C_s - T_s = 546.1 + 223 - 120 = 649.1 \text{ kip}$$

$$\phi P_n = 0.65 \times 649 = 421.9 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 546.1 \times \left(\frac{16}{2} - \frac{8.03}{2}\right) + 223 \times \left(\frac{16}{2} - 2.5\right) + 120 \times \left(13.50 - \frac{16}{2}\right) = 338.54 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 339 = 220.05 \text{ kip.ft}$$

## 4. Bar Stress Near Tension Face Equal to $f_y$, ($f_s = f_y$)

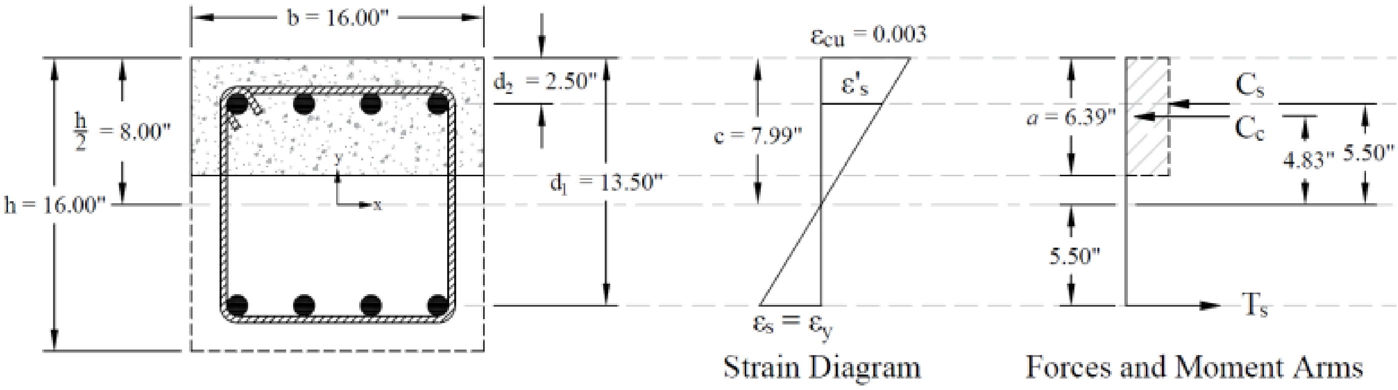



Figure 5 – Strains, Forces, and Moment Arms ($f_s = f_y$)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for ϕ for columns in which ϕ increases from 0.65 (or 0.75 for spiral columns) up to 0.90.

### 4.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

$$\varepsilon_s = \varepsilon_y = 0.00207 \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00207 + 0.003} \times 0.003 = 7.99 \text{ in.}$$

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

***ACI 318-19 (22.2.2.4.2)***

$$a = \beta_1 \times c = 0.80 \times 7.99 = 6.39 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (7.99 - 2.5) \times \frac{0.003}{7.99} = 0.00206 \text{ (Compression)} < \varepsilon_y$$

### 4.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5{,}000 \times 6.39 \times 16 = 434.6 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = f_y = 60{,}000 \text{ psi}$$

$$\text{T}_s = f_y \times A_{s1} = 60{,}000 \times 4 = 240 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f_s' = \varepsilon_s' \times E_s = 0.00206 \times 29{,}000{,}000 = 59{,}778 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ($ab$) used to compute $C_c$. As a result, it is necessary to subtract $0.85f_c$' from $f_s$' before computing $C_s$:

$$\text{C}_s = (f_s' - 0.85 f_c') \times A_{s2} = (59{,}778 - 0.85 \times 5{,}000) \times 4 = 222.1 \text{ kip}$$

### 4.3. $\phi P_n$ and $\phi M_n$

$$P_n = C_c + C_s - T_s = 434.6 + 222.1 - 240 = 416.8 \text{ kip}$$

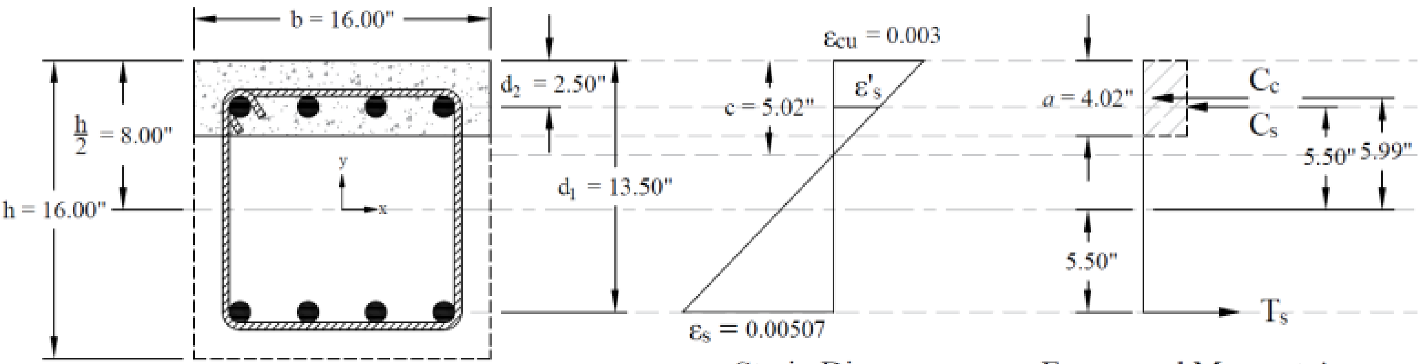
$$\phi P_n = 0.65 \times 417 = 270.9 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 434.6 \times \left(\frac{16}{2} - \frac{6.39}{2}\right) + 222.1 \times \left(\frac{16}{2} - 2.5\right) + 240 \times \left(13.50 - \frac{16}{2}\right) = 385.81 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 386 = 250.77 \text{ kip.ft}$$

## 5. Bar Strain Near Tension Face Equal to $\varepsilon_y$ + 0.003, ($\varepsilon_s$ = 0.00507 in./in.)




Strain Diagram Forces and Moment Arms

5.1. c, $a$, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$\varepsilon_s = \varepsilon_y + 0.003 = 0.00207 + 0.003 = 0.00507 > \varepsilon_y \rightarrow$ tension reinforcement has yielded

$\therefore \phi = 0.9$ **_ACI 318-19 (Table 21.2.2)_**

$\varepsilon_{cu} = 0.003$ **_ACI 318-19 (22.2.2.1)_**

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00507 + 0.003} \times 0.003 = 5.02 \text{ in.}$$

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

**_ACI 318-19 (22.2.2.4.2)_**

$a = \beta_1 \times c = 0.80 \times 5.02 = 4.02$ in. **_ACI 318-19 (22.2.2.4.1)_**

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ **_ACI 318-19 (Table 22.2.2.4.3)_**

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (5.02 - 2.5) \times \frac{0.003}{5.02} = 0.00151 \text{ (Compression)} < \varepsilon_y$$

5.2. Forces in the concrete and steel

$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5{,}000 \times 4.02 \times 16 = 273.1 \text{ kip}$ ***ACI 318-19 (22.2.2.4.1)***

$f_s = f_y = 60{,}000 \text{ psi}$

$\text{T}_s = f_y \times A_{s1} = 60{,}000 \times 4 = 240 \text{ kip}$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$\therefore f_s' = \varepsilon_s' \times E_s = 0.00151 \times 29{,}000{,}000 = 43{,}667 \text{ psi}$

The area of the reinforcement in this layer has been included in the area ($ab$) used to compute $C_c$. As a result, it is necessary to subtract $0.85f_c'$ from $f_s'$ before computing $C_s$:

$\text{C}_s = (f_s' - 0.85 f_c') \times A_{s2} = (43{,}667 - 0.85 \times 5{,}000) \times 4 = 157.7 \text{ kip}$

5.3. $\phi P_n$ and $\phi M_n$

$P_n = C_c + C_s - T_s = 273.1 + 157.7 - 240 = 190.7 \text{ kip}$

$\phi P_n = 0.90 \times 191 = 171.6 \text{ kip}$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 273.1 \times \left(\frac{16}{2} - \frac{4.02}{2}\right) + 157.7 \times \left(\frac{16}{2} - 2.5\right) + 240 \times \left(13.50 - \frac{16}{2}\right) = 318.61 \text{ kip.ft}$$

$\phi M_n = 0.90 \times 320 = 286.75 \text{ kip.ft}$

## 6. Pure Bending

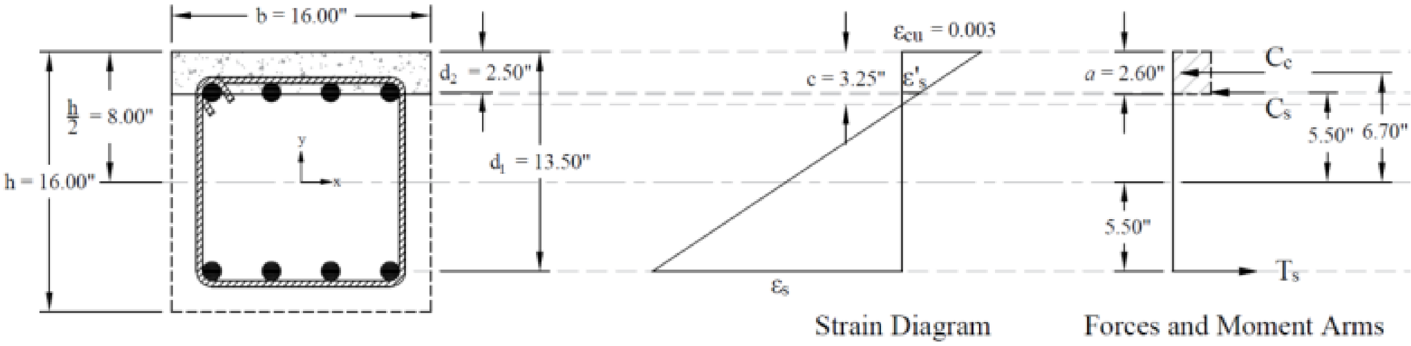



Figure 7 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the nominal axial load capacity, $P_n$, is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

### 6.1. c, a, and strains in the reinforcement

Try $c = 3.25$ in.

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

***ACI 318-19 (22.2.2.4.2)***

$a = \beta_1 \times c = 0.80 \times 3.25 = 2.60$ in. ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.003}{c} = (13.50 - 3.25) \times \frac{0.003}{3.25} = 0.00946 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$\therefore \phi = 0.9$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (3.25 - 2.5) \times \frac{0.003}{3.25} = 0.00069 \text{ (Compression)} < \varepsilon_y$$

### 6.2. Forces in the concrete and steel

$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5,000 \times 2.6 \times 16 = 176.8$ kip ***ACI 318-19 (22.2.2.4.1)***

$f_s = f_y = 60,000$ psi

$$T_s = f_y \times A_{s1} = 60{,}000 \times 4 = 240 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f_s' = \varepsilon_s' \times E_s = 0.00069 \times 29{,}000{,}000 = 20{,}077 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ($ab$) used to compute $C_c$. As a result, it is necessary to subtract $0.85f_c$' from $f_s$' before computing $C_s$:

$$C_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(20{,}077 - 0.85 \times 5{,}000\right) \times 4 = 63.3 \text{ kip}$$

6.3. <u>$\phi P_n$ and $\phi M_n$</u>

$$P_n = C_c + C_s - T_s = 176.8 + 63.3 - 240 \approx 0 \text{ kip} \rightarrow \phi P_n \approx 0 \text{ kip}$$

The assumption that c = 3.25 in. is correct

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 176.8 \times \left(\frac{16}{2} - \frac{2.60}{2}\right) + 63.3 \times \left(\frac{16}{2} - 2.5\right) + 240 \times \left(13.50 - \frac{16}{2}\right) = 237.73 \text{ kip.ft}$$

$$\phi M_n = 0.90 \times 238 = 213.96 \text{ kip.ft}$$

## 7. Maximum Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

7.1. <u>$P_{nt}$ and $\phi P_{nt}$</u>

$$P_{nt} = f_y \times \left(A_{s1} + A_{s2}\right) = 60{,}000 \times (4 + 4) = 480.0 \text{ kip}$$ ***<u>ACI 318-19 (22.4.3.1)</u>***

$$\phi = 0.9$$ ***<u>ACI 318-19 (Table 21.2.2)</u>***

$$\phi P_{nt} = 0.90 \times 480.0 = 432.0 \text{ kip}$$

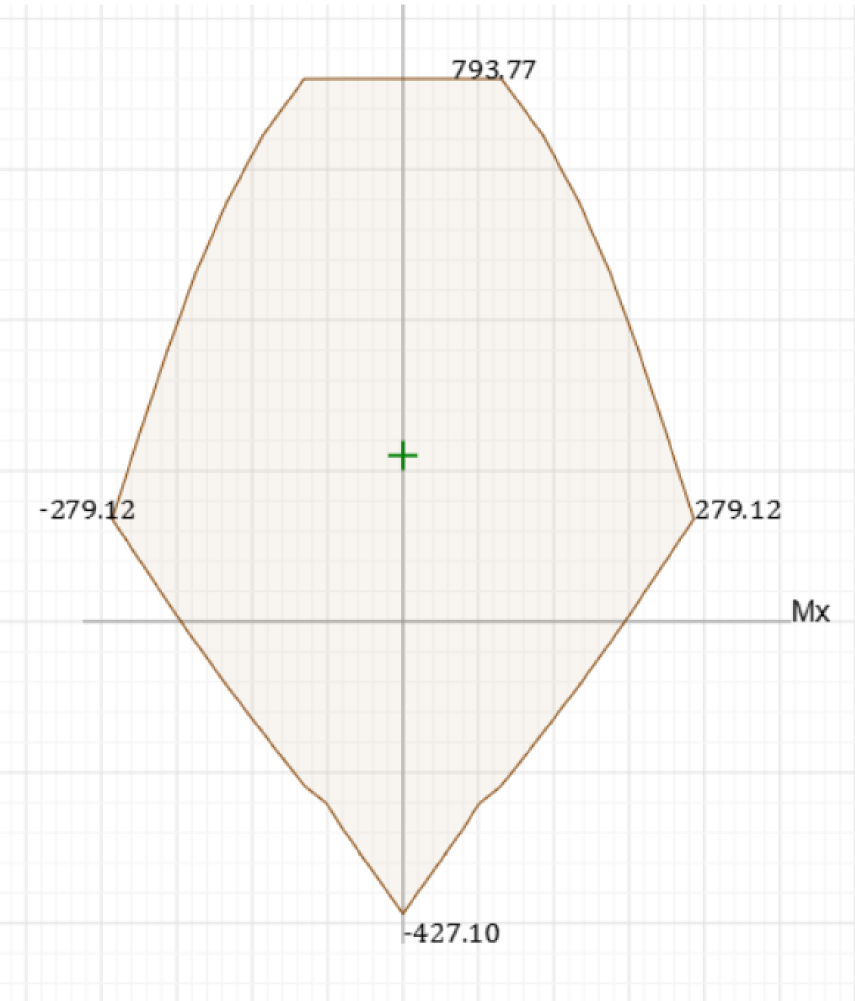
7.2. <u>$M_n$ and $\phi M_n$</u>

Since the section is symmetrical

$$M_n = \phi M_n = 0.00 \text{ kip.ft}$$

| | ϕ Pn (kip) | | | | ϕ $M_n$ (kip.ft) | | | |
|---|---|---|---|---|---|---|---|---|
| | **Manual** | **Software A** | **TQS** | **Dif** | **Manual** | **Software A** | **TQS** | **Dif** |
| Compressão Máxima | 997,10 | 997,10 | 966,54 | 3% | 0,00 | 0,00 | 0,00 | - |
| Compr. Max. Permitida | 797,70 | 797,70 | 793,77 | 0% | - | - | - | - |
| $f_s$=0 | 622,30 | 622,30 | 622,30 | 0% | 169,86 | 170,00 | 172,50 | 2% |
| $f_s$=0,5 $f_y$ | 421,90 | 421,90 | 421,90 | 0% | 220,05 | 220,00 | 217,50 | -1% |
| Ponto de Balance | 270,90 | 270,90 | 270,90 | 0% | 250,77 | 251,00 | 254,00 | 1% |
| Momento Máximo | 171,60 | 171,60 | 171,60 | 0% | 286,75 | 286,00 | 279,12 | -3% |
| Flexão Pura | 0,00 | 0,00 | 0,00 | - | 213,96 | 214,00 | 218,00 | 2% |
| Tração Máxima | 432,00 | 432,00 | 426,95 | 1% | 0,00 | 0,00 | 0,00 | - |

Curva de Interação TQS (kip, kip.ft)




*Conclusão:*

Os valores apresentados nos diagramas são muito semelhantes com diferenças mínimas.